Problem Set 9

Macroeconomics III

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Problem 1

Two firms with zero marginal cost produce imperfect substitutes. Demand for firm i is given by:

$$y_i = a - bp_i + cp_j$$

a) Show that, in the absence of adjustment costs, the (Bertrand-Nash) equilibrium is given by $p_1 = p_2 = \frac{a}{2b-c}$. Note that given the price quoted by the other firm, each firm's profit is perfectly symmetric with respect to its own price.

$$\begin{aligned}\pi_i &= p_i \cdot y_i \\ &= p_i(a - bp_i + cp_j)\end{aligned}$$

1. Solve for optimal price p_i given p_j
2. Impose identical prices, $p_1 = p_2 = \bar{p}$, as firms are identical and isolate $\bar{p}$

Problem 1a - Bertrand-Nash Equilibrium

Show that, in the absence of adjustment costs, the (Bertrand-Nash) equilibrium is given by $p_1 = p_2 = \frac{a}{2b-c}$. Note that given the price quoted by the other firm, each firm's profit is perfectly symmetric with respect to its own price.

The FOC of the profits wrt. the price is:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \quad \implies \quad a - 2bp_i + cp_j = 0$$

$$p_i = \frac{a + cp_j}{2b}$$

Firms are identical and the expression for p_i is identical to that of p_j, why $p_1 = p_2 = \bar{p}$.We then find $\bar{p}$:

$$\bar{p} = \frac{a + c\bar{p}}{2b}$$

$$2b\bar{p} = a + c\bar{p}$$

$$\bar{p} = \frac{a}{2b - c}$$

Problem 1b - Don't Adjust NE (1/3)

Compute the set of values of a (around a^*) for which not to adjust prices is a Nash equilibrium.

First, we find the profits of firm i from adjusting while firm j doesn't adjust:

$$
\begin{aligned}
\pi_i^A(j = DA) &= p_i(a - bp_i + cp_j) - k \\
&= \underbrace{\frac{a + c\bar{p}}{2b}}_{p_i}\left(a - b\underbrace{\frac{a + c\bar{p}}{2b}}_{p_i} + c\underbrace{\bar{p}}_{p_j = \bar{p}}\right) - k \\
&= \frac{a + c\bar{p}}{2b} \cdot \left(a - \frac{a + c\bar{p}}{2} + c\bar{p}\right) - k \\
&= \frac{a + c\bar{p}}{2b} \cdot \frac{2a - a - c\bar{p} + 2c\bar{p}}{2} - k \\
&= \frac{a + c\bar{p}}{2b} \cdot \frac{a + c\bar{p}}{2} - k \\
&= \frac{(a + c\bar{p})^2}{4b} - k
\end{aligned}
$$

Problem 1b - Don't Adjust NE (2/3)

Then we find the profits if firm i doesn't adjust:

$$\begin{aligned}\pi_i^{DA}(j = DA) &= p_i(a - bp_i + cp_j) \\ &= \bar{p}(a - b\bar{p} + c\bar{p}) \\ &= a\bar{p} - b\bar{p}^2 + c\bar{p}^2\end{aligned}$$

Finally, we find the values of a where $\pi_i^{DA} > \pi_i^A$,

$$\begin{aligned}\pi_i^{DA} &> \pi_i^A \\ a\bar{p} - b\bar{p}^2 + c\bar{p}^2 &> \frac{(a + c\bar{p})^2}{4b} - k \\ k &> \frac{(a + c\bar{p})^2}{4b} - a\bar{p} + b\bar{p}^2 - c\bar{p}^2 \\ 4bk &> (a + c\bar{p})^2 - 4ba\bar{p} + 4b^2\bar{p}^2 - 4bc\bar{p}^2\end{aligned}$$

We then rewrite the right hand side.

Problem 1b - Don't Adjust NE (3/3)

$$4bk > (a + c\bar{p})^2 - 4ba\bar{p} + 4b^2\bar{p}^2 - 4bc\bar{p}^2$$
$$4bk > a^2 + c^2\bar{p}^2 + 2ac\bar{p} - 4ba\bar{p} + 4b^2\bar{p}^2 - 4bc\bar{p}^2$$
$$4bk > a^2 - 2a\bar{p}(2b - c) + \bar{p}^2(c^2 + 4b^2 - 4bc)$$
$$4bk > a^2 - 2a\bar{p}(2b - c) + (\bar{p}(2b - c))^2$$

We now use that $\bar{p} = \frac{a^*}{2b-c}$

$$4bk > a^2 - 2a\frac{a^*}{2b - c}(2b - c) + \left(\frac{a^*}{2b - c}(2b - c)\right)^2$$
$$4bk > a^2 - 2aa^* + (a^*)^2$$
$$4bk > (a - a^*)^2$$

By taking the square root on both sides, we see that not adjusting prices is a Nash-Equilibrium when

$$2\sqrt{bk} > |a - a^*|$$

The interval is symmetric around a^* and increasing in k.

Problem 1c - Adjusting NE (1/3)

Compute the set of values of a (around a^*) for which to adjust prices is a Nash equilibrium.

Next, we focus on the NE where both firms adjust. Profits from adjusting is given by:

$$\begin{aligned}
\pi_i^A(j = A) &= p_i(a - bp_i + cp_j) - k \\
&= \underbrace{\frac{a}{2b - c}}_{p_i}\left(a + (c - b)\underbrace{\frac{a}{2b - c}}_{p_i = p_j}\right) - k \\
&= \frac{a^2}{2b - c}\left(1 + (c - b)\frac{1}{2b - c}\right) - k \\
&= \frac{a^2}{2b - c} \cdot \frac{2b - c + c - b}{2b - c} - k \\
&= \frac{a^2}{2b - c} \cdot \frac{b}{2b - c} - k \\
&= \frac{ba^2}{(2b - c)^2} - k
\end{aligned}$$

Problem 1c - Adjusting NE (2/3)

Profits from not adjusting is given by:

$$\begin{aligned}
\pi_i^{DA}(j=A) &= p_i(a - bp_i + cp_j) \\
&= \frac{a^*}{2b-c}\left(a - b\frac{a^*}{2b-c} + c\frac{a}{2b-c}\right) \\
&= \frac{a^*}{2b-c}\cdot\frac{2ab - ac - ba^* + ca}{2b-c} \\
&= \frac{a^*}{2b-c}\cdot\frac{2ab - ba^*}{2b-c} \\
&= \frac{ba^*(2a-a^*)}{(2b-c)^2}
\end{aligned}$$

Hence, the condition is given by,

$$\pi_i^{A}(j=A) > \pi_i^{DA}(j=A)$$

$$\frac{ba^2}{(2b-c)^2} - k > \frac{ba^*(2a-a^*)}{(2b-c)^2}$$

Problem 1c - Adjusting NE (3/3)

We continue by rewriting the condition:

$$\frac{ba^2}{(2b-c)^2} - k > \frac{ba^*(2a-a^*)}{(2b-c)^2}$$

$$\frac{ba^2 - ba^*(2a-a^*)}{(2b-c)^2} > k$$

$$a^2 + (a^*)^2 - 2aa^* > k\frac{(2b-c)^2}{b}$$

$$(a-a^*)^2 > k\frac{(2b-c)^2}{b} \tag{1}$$

Taking the square root on both side yields the following condition:

$$(2b-c)\sqrt{\frac{k}{b}} < |a-a^*|$$

Note: It is possible that the solutions overlap when $c > 0$

$$(2b-c)\sqrt{\frac{k}{b}} < |a-a^*| < 2\sqrt{bk}$$

Problem 1d - Price Sensitivity

If only Pareto optimal equilibria are observed, discuss the following statement: "Duopolistic prices are more sensitive to positive shocks than to negative shocks of the same magnitude"

The profits when both firms adjust are,

$$\begin{aligned}
\pi^A(j = A) &= \frac{a}{2b-c}\left(a - b\frac{a}{2b-c} + c\frac{a}{2b-c}\right) - k \\
&= \frac{ba^2}{(2b-c)^2} - k
\end{aligned}$$

The profits when both firms don't adjust are,

$$\begin{aligned}
\pi^{DA}(j = DA) &= \frac{a*}{2b-c}\left(a - b\frac{a*}{2b-c} + c\frac{a*}{2b-c}\right) \\
&= \frac{aa^*}{2b-c} + (c-b)\frac{(a^*)^2}{(2b-c)^2}
\end{aligned}$$

Problem 1d - Price sensitivity

Hence, it is pareto efficient to not adjust when

$$\frac{aa^*}{2b-c} + (c-b)\frac{(a^*)^2}{(2b-c)^2} > \frac{ba^2}{(2b-c)^2} - k$$

$$aa^*(2b-c) + (c-b)(a^*)^2 > ba^2 - k(2b-c)^2$$

$$2aa^* - \frac{c}{b}aa^* + \frac{c}{b}(a^*)^2 - (a^*)^2 > a^2 - \frac{(2b-c)^2}{b}k$$

Combining on the right hand side yields:

$$0 > \underbrace{a^2 - 2aa^* + (a^*)^2 - \frac{(2b-c)^2}{b}k}_{>0 \text{ from } (1)} + \frac{c}{b}a^*(a-a^*)$$

Hence, for not adjusting to be pareto efficient, the shock must be sufficiently negative, $a < a^*$.

The statement that we have to evaluate is, therefore, true. In the region where the NE of ajusting overlaps the NE of not adjusting, the shock must be negative for the firms to not adjust.

Problem 2

We consider a model with price setting under imperfect competition. Firms predetermine a price for two periods based on demand and pricing of competitors.

$$y_t = m_t - p_t \tag{2}$$

$$p_t = \frac{1}{2}\left(p_t^1 + p_t^2\right) \tag{3}$$

$$p_t^i = b\mathbb{E}_{t-i}[m_t] + (1-b)\mathbb{E}_{t-i}[p_t] \tag{4}$$

The timing is as follows

- Firm 2 sets their prices between $t-2$ and $t-1$. p^2 is then fixed until after period t and determines the price level p_{t-1}
- Demand m_{t-1} and output y_{t-1} are determined.
- Firm 1 sets their prices between $t-1$ and t. p^1 is then fixed until after period $t+1$ and determines the price level p_t
- Demand m_t and output y_t are determined.

Problem 2a - p_t^1 as a function of m_t

Find $\mathbb{E}_{t-2}[p_t^1]$ as a function of m_t and expectations on m_t *(And of p_t^2)*

$$\begin{aligned}
\mathbb{E}_{t-2}[p_t^1] &= \mathbb{E}_{t-2}\left[b\mathbb{E}_{t-1}[m_t] + (1-b)\mathbb{E}_{t-1}[p_t]\right] \\
&= b\mathbb{E}_{t-2}[m_t] + (1-b)\mathbb{E}_{t-2}[p_t] \\
&= b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}\mathbb{E}_{t-2}[p_t^1] + \frac{1-b}{2}\mathbb{E}_{t-2}[p_t^2]
\end{aligned}$$

Prices of firm 2 is known at $t-2$, so $\mathbb{E}_{t-2}[p_t^2] = p_t^2$

$$\begin{aligned}
&= b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}\mathbb{E}_{t-2}[p_t^1] + \frac{1-b}{2}p_t^2 \\
\mathbb{E}_{t-2}[p_t^1]\left(1 - \frac{1-b}{2}\right) &= b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}p_t^2 \\
\mathbb{E}_{t-2}[p_t^1]\frac{1+b}{2} &= b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}p_t^2 \\
\mathbb{E}_{t-2}[p_t^1] &= \frac{2b}{1+b}\mathbb{E}_{t-2}[m_t] + \frac{1-b}{1+b}p_t^2 \\
&= p_t^2 + \frac{2b}{1+b}(\mathbb{E}_{t-2}[m_t] - p_t^2)
\end{aligned}$$

Problem 2b - Find p_t^2 (1/2)

Similarly find p_t^1 and p_t^2

We start by finding p_t^2,

$$p_t^2 = b\mathbb{E}_{t-2}[m_t] + (1-b)\mathbb{E}_{t-2}[p_t]$$
$$p_t^2 = b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}\mathbb{E}_{t-2}[p_t^1] + \frac{1-b}{2}\mathbb{E}_{t-2}[p_t^2]$$
$$p_t^2 = b\mathbb{E}_{t-2}[m_t] + \frac{1-b}{2}\underbrace{\left[p_t^2 + \frac{2b}{1+b}\left(\mathbb{E}_{t-2}[m_t] - p_t^2\right)\right]}_{\mathbb{E}_{t-2}[p_t^1]} + \frac{1-b}{2}[p_t^2]$$
$$p_t^2 = b\mathbb{E}_{t-2}[m_t] + (1-b)p_t^2 + \frac{b(1-b)}{1+b}\left(\mathbb{E}_{t-2}[m_t] - p_t^2\right)$$

Problem 2b - Find p_t^2 (2/2)

We continue by rewriting the right hand side

$$
\begin{aligned}
p_t^2 &= b\mathbb{E}_{t-2}[m_t] + (1-b)p_t^2 + \frac{b(1-b)}{1+b}\left(\mathbb{E}_{t-2}[m_t] - p_t^2\right) \\
0 &= b\mathbb{E}_{t-2}[m_t] - bp_t^2 + \frac{b(1-b)}{1+b}\left(\mathbb{E}_{t-2}[m_t] - p_t^2\right) \\
0 &= \left(\mathbb{E}_{t-2}[m_t] - p_t^2\right)\underbrace{\left(b + \frac{b(1-b)}{1+b}\right)}_{>0}
\end{aligned}
$$

Hence, we get the following result,

$$p_t^2 = \mathbb{E}_{t-2}[m_t]$$

Problem 2b - Find p_t^1

Similarly find p_t^1 and p_t^2

$$\begin{aligned}
p_t^1 &= b\mathbb{E}_{t-1}[m_t] + (1-b)\mathbb{E}_{t-1}[p_t] \\
&= b\mathbb{E}_{t-1}[m_t] + \frac{1-b}{2}p_t^1 + \frac{1-b}{2}p_t^2 \\
p_t^1\left(1 - \frac{1-b}{2}\right) &= b\mathbb{E}_{t-1}[m_t] + \frac{1-b}{2}p_t^2 \\
p_t^1\frac{1+b}{2} &= b\mathbb{E}_{t-1}[m_t] + \frac{1-b}{2}p_t^2 \\
p_t^1 &= \frac{2b}{1+b}\mathbb{E}_{t-1}[m_t] + \frac{1-b}{1+b}p_t^2 \\
p_t^1 &= p_t^2 + \frac{2b}{1+b}(\mathbb{E}_{t-1}[m_t] - p_t^2)
\end{aligned}$$

Or equivalently,

$$p_t^1 = \mathbb{E}_{t-2}[m_t] + \frac{2b}{1+b}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)$$

Problem 2c - Find p_t and y_t

Find equilibrium p_t and y_t as functions of m_t and expectations on m_t.

We find the price level by inserting our previous results,

$$
\begin{aligned}
p_t &= \frac{1}{2}(p_t^1 + p_t^2) \\
&= \frac{1}{2}\left(\underbrace{\mathbb{E}_{t-2}[m_t] + \frac{2b}{1+b}\Big(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\Big)}_{p_t^1} + \underbrace{\mathbb{E}_{t-2}[m_t]}_{p_t^2}\right) \\
&= \mathbb{E}_{t-2}[m_t] + \frac{b}{1+b}(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t])
\end{aligned}
$$

We find output by inserting the price-level,

$$
\begin{aligned}
y_t &= m_t - p_t \\
&= m_t - \mathbb{E}_{t-2}[m_t] - \frac{b}{1+b}(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]) \\
&= \frac{1}{1+b}\left(m_t - \mathbb{E}_{t-2}[m_t]\right) + \frac{b}{1+b}\left(m_t - \mathbb{E}_{t-1}[m_t]\right)
\end{aligned}
$$

Problem 2e - Optimal Monetary Policy - Output (1/3)

We add output fluctuations to the model in order to analyse the role of monetary policy in stabilizing output. Aggregate demand is now given by,

$$y_t = m_t - p_t + v_t$$
$$v_t = v_{t-1} + \epsilon_t$$
$$m_t = a_1 \epsilon_{t_1} + a_2 \epsilon_{t-2}$$

Find the values of a_i that minimizes output volatility. (you can replace m by $m + v$ in the previous solution). Why monetary policy is able to reduce output volatility? Interpret.

We rewrite the stochastic money velocity.

$$v_t = v_{t-1} + \epsilon_t = v_{t-2} + \epsilon_t + \epsilon_{t-1} = v_{t-3} + \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2}$$

This allows us to rewrite $m_t + v_t$ such that,

$$\begin{aligned} m_t + v_t &= a_1 \epsilon_{t-1} + a_2 \epsilon_{t-2} + v_{t-3} + \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} \\ &= v_{t-3} + \epsilon_t + (1 + a_1)\epsilon_{t-1} + (1 + a_2)\epsilon_{t-2} \end{aligned}$$

Problem 2e - Optimal Monetary Policy - output (2/3)

We then find the expectations while using $\mathbb{E}_t[\epsilon_{t+1}] = 0$,

$$\begin{aligned}
m_t + v_t &= v_{t-3} + \epsilon_t + (1 + a_1)\epsilon_{t-1} + (1 + a_2)\epsilon_{t-2} \\
\mathbb{E}_{t-1}[m_t + v_t] &= v_{t-3} + (1 + a_1)\epsilon_{t-1} + (1 + a_2)\epsilon_{t-2} \\
\mathbb{E}_{t-2}[m_t + v_t] &= v_{t-3} + (1 + a_2)\epsilon_{t-2}
\end{aligned}$$

The differences are thus,

$$\begin{aligned}
m_t + v_t - \mathbb{E}_{t-1}[m_t + v_t] &= \epsilon_t \\
m_t + v_t - \mathbb{E}_{t-2}[m_t + v_t] &= \epsilon_t + (1 + a_1)\epsilon_{t-1}
\end{aligned}$$

We insert this into our equation for the output,

$$\begin{aligned}
y_t &= \frac{1}{1+b}\left(m_t + v_t - \mathbb{E}_{t-2}[m_t + v_t]\right) + \frac{b}{1+b}\left(m_t + v_t - \mathbb{E}_{t-1}[m_t + v_t]\right) \\
&= \frac{1}{1+b}\epsilon_t + \frac{1}{1+b}(1 + a_1)\epsilon_{t-1} + \frac{b}{1+b}\epsilon_t \\
&= \epsilon_t + \frac{1}{1+b}(1 + a_1)\epsilon_{t-1}
\end{aligned}$$

Problem 2e - Optimal Monetary Policy - output (3/3)

Thus, the output is given by:

$$y_t = \epsilon_t + \frac{1}{1+b}(1+a_1)\epsilon_{t-1}$$

The variance of the output is,

$$var(y_t) = var(\epsilon_t) + \frac{1}{(1+b)^2}(1+a_1)^2 var(\epsilon_{t-1})$$

Since the variance can never be negative, the volatility minimzing value of a_1 is -1,

$$a_1 = -1$$

The optimal policy for output stabilization does not include a reaction to ϵ_{t-2}. This is because all firms have observed ϵ_{t-2}, so output will have adjusted itself in response to shocks from two periods ago.

Problem 2f - Optimal Monetary Policy - Prices (1/2)

What are the values of a_i that minimizes price volatility? Is there a conflict between the goals of price and output stabilization? Interpret.

The price level is given by:

$$p_t = \mathbb{E}_{t-2}[m_t + v_t] + \frac{b}{1+b}(\mathbb{E}_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t])$$

We use our results from 2e

$$\begin{aligned}
\mathbb{E}_{t-1}[m_t + v_t] &= v_{t-3} + (1 + a_1)\epsilon_{t-1} + (1 + a_2)\epsilon_{t-2} \\
\mathbb{E}_{t-2}[m_t + v_t] &= v_{t-3} + (1 + a_2)\epsilon_{t-2} \\
\mathbb{E}_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t] &= (1 + a_1)\epsilon_{t-1}
\end{aligned}$$

The price level is thus,

$$p_t = v_{t-3} + (1 + a_2)\epsilon_{t-2} + \frac{b}{1+b}(1 + a_1)\epsilon_{t-1}$$

Problem 2f - Optimal Monetary Policy - Prices (2/2)

The price level is given by

$$p_t = v_{t-3} + (1 + a_2)\epsilon_{t-2} + \frac{b}{1+b}(1 + a_1)\epsilon_{t-1}$$

The variance of the price level is then:

$$var(p_t) = var(v_{t-3}) + (1 + a_2)^2 var(\epsilon_{t-2}) + \left(\frac{b(1 + a_1)}{(1 + b)}\right)^2 var(\epsilon_{t-1})$$

The optimal values are thus, $a_1 = -1$ and $a_2 = -1$. For price stability the monetary response is a function of both ϵ_{t-1} and ϵ_{t-2}. This is because, the output adjustment from question 2e was achieved by adjusted prices, which we want to avoid in this question.

Is there a conflict between price and output stabilization?

No. There is no trade-off between price and output stabilization in this economy.